

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 9

- If a, b, c are in G.P, b is the GM between a & c , $b^2 = ac$, therefore $b = \sqrt{ac}$; $a > 0, c > 0$
- If a, b are two given numbers & $a, G_1, G_2, \dots, G_n, b$ are in G.P, then $G_1, G_2, G_3 \dots G_n$ are n GMs between a & b .

$$G_1 = a \cdot \left(\frac{b}{a}\right)^{\frac{1}{n+1}} G_2 = a \cdot \left(\frac{b}{a}\right)^{\frac{2}{n+1}} \dots G_n = a \cdot \left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

or

$$G_1 = ar, G_2 = ar^2, \dots G_n = ar^{n-1}$$

$$\text{where } r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

A sequence is said to be a geometric progression or G.P, if the ratio of any term to its preceding term is same throughout. This constant factor is called 'Common ratio'. Usually we denote the first term of a G.P by ' a ' and its common ratio by ' r '. The general or n^{th} term of G.P is given by $a_n = ar^{n-1}$. The sum S_n of the first n terms of G.P is given by

$$S_n = \frac{a(r^n - 1)}{r - 1} \text{ or } \frac{a(1 - r^n)}{1 - r} \text{ if } r \neq 1$$

Sum of infinite terms of G.P is given by

$$S_n = \frac{a}{1 - r}; |r| < 1$$

Sequence is a function whose domain is the set of N natural numbers.

Real sequence: A sequence whose range is a subset of R is called a real sequence

Series: If $a_1, a_2, a_3, \dots, a_n$ is a sequence, then the expression $a_1 + a_2 + a_3 + \dots + a_n + \dots$ is a series. A series is finite or infinite according to the no. of terms in the corresponding sequence as finite or infinite.

Progression: Those sequences whose terms follow certain patterns are called progression.

In a series, where each term is formed by multiplying the corresponding term of an AP & GP is called Arithmetico-Geometric Series.

$$\text{Eg: } 1 + 3x + 5x^2 + 7x^3 + \dots$$

Here, $1, 3, 5, \dots$ are in AP and $1, x, x^2, \dots$ are in GP.

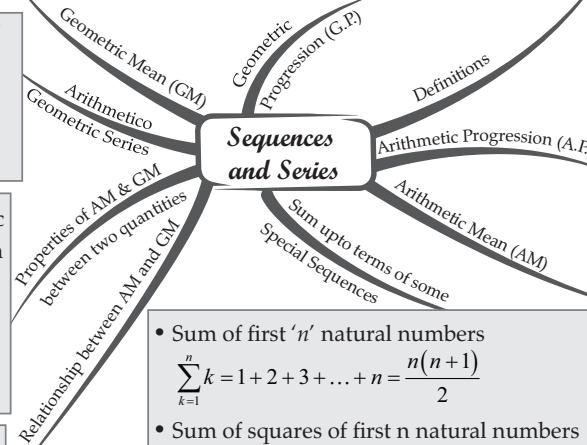
- If A and G are respectively arithmetic and geometric means between two positive numbers a and b ; then the quadratic equation have a, b as its root is $x^2 - 2Ax + G^2 = 0$
- If A and G be the AM & GM between two positive numbers, then the numbers are $A \pm \sqrt{A^2 - G^2}$

Let A and G be the AM and GM of two given positive real numbers a & b , respectively. Then,

$$A = \frac{a+b}{2} \text{ and } G = \sqrt{ab}$$

$$\text{Thus, we have, } A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2} = \frac{(\sqrt{a}-\sqrt{b})^2}{2} \geq 0 \dots\dots\dots (1)$$

From (1), we obtain the relationship $A \geq G$



An A.P is a sequence in which terms increase or decrease regularly by the fixed number (same constant). This fixed number is called 'common difference' of the A.P. If ' a ' is the first term & ' d ' is the common difference and ' l ' is the last term of A.P, then general term or the n^{th} term of the A.P is given by $a_n = a + (n-1)d$.

The sum S_n of the first n terms of an A.P is given by

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l]$$

If three numbers are in A.P, then the middle term is called AM between the other two, so if a, b, c , are in A.P, b is AM of a and c .

AM for any ' n ' +ve numbers $a_1, a_2, a_3, \dots, a_n$ is

$$AM = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$$

n-Arithmetic Mean between Two Numbers: If a, b are any two given numbers & $a, A_1, A_2, \dots, A_n, b$ are in A.P then A_1, A_2, \dots, A_n are n AM's between a & b .

$$A_1 = a + \frac{b-a}{n+1}, A_2 = a + \frac{2(b-a)}{n+1} \dots A_n = a + \frac{n(b-a)}{n+1}$$

or $A_1 = a + d, A_2 = a + 2d, \dots, A_n = a + nd$, where $d = \frac{b-a}{n+1}$

